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COLLABORATIVE OPTIMIZATION OF DIFFERENTIAL STEERING FOR IN-WHEEL ELECTRIC VEHICLE*

W. Z. ZHAO[1**], C. Y. Wang[2] AND Z. Q. ZHANG[3]
[1,2,3]Dept. of Vehicle Eng., Nanjing University of Aeronautics and Astronautics, P. R. China
[1,2]Dept. of Mechanical Eng., University of Michigan, USA
Email: zhaowanzhong@126.com

Abstract– Differential steering of in-wheel electric vehicle provides the functions of both active steering and power assisted steering with the coupling control of force and displacement transfer characteristic of system. A collaborative optimization model of the differential power-assisted steering system of in-wheel electric vehicle is built, with steering economy as the main system optimization goal, steering road feel, steering sensitivity and torque sensor performance as the subsystem optimization goals. Considering the coupled relationship of each discipline, the main system is optimized by the particle swarm algorithm, and the subsystems are optimized by the directional heuristic search algorithm which is good on local optimization. The simulation results show that the collaborative optimization based on particle swarm algorithm has more optimal solution sets and fast convergence by considering the coupling relationship between different disciplines, and the comprehensive performance of in-wheel electric vehicle is improved.

Keywords– In-wheel electric vehicle, differential steering, collaborative optimization, particle swarm algorithm

1. INTRODUCTION

Because of the continuous depletion of world energy resources and growing environmental pollution, electric vehicles, as environment friendly, energy saving and quiet vehicles, cause research upsurge at home and abroad. An important direction of the new generation of electric vehicles, in-wheel electric vehicle is a cause for concern with its small vehicle equipment quality, high power transmission efficiency. With independent driving of the in-wheel motor, differential steering system (DSS) can realize power steering function by changing the left and right wheel motor output torque to control the force transmission characteristics and active steering function by providing additional corner of wheel motors to control displacement transmission characteristics[1, 2]. Therefore, the development of DSS based on in-wheel electric vehicle can not only combine the steering portability and the steering road feel perfectly but also unite the safety and economy. As an ideal steering technology, the DSS owns perfect application prospect and potential technical development.

At present, the study focused primarily on modeling and simulation of electric wheel motors, torque coordination control, etc. Research on the optimization of differential steering was very little [3]. Reports of DSS optimization are hard to search. The optimization of the DSS involves not only steering road feel, steering sensitivity and steering stability, but also the steering economy and some other disciplines with coupled affection. Therefore, the optimization of the DSS is essentially a problem of multidisciplinary design optimization (MDO) [4]. The DSS will get the best performance only if each discipline is considered comprehensively and put together to optimize collaboratively. MDO is a methodology of designing complicated system and subsystem by fully exploiting and exploring the collaborative

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**Corresponding author

mechanism in every system. The optimal solution in system level can be realized by coordinating the coupled relationship of each subsystem [5, 6].

Without evolution operator operation such as cross, variation and selection in other evolution algorithm, Particle swarm optimization algorithm (PSO) regards individuals in the swarm as particles without quality and volume in D-dimensional search space. Each particle gathers around its own best known position and the entire swarm's best known position with certain speed in the search space so that the optimal solution will be achieved [7]. Due to the good biological social background and the need for fewer parameters, it is easy to understand and implement. PSO has been widespread in the scientific research and engineering practice for its strong global search capability for multimodal nonlinear.

In this paper, particle swarm algorithm is applied to optimize the DSS based on the multidisciplinary collaborative optimization. The research work provides a theoretical foundation for the design of the DSS and the in-wheel electric vehicle.

2. MECHANISM OF DSS

The driving force and steering assist torque are provided by two in-wheel motors of DSS. The output torques of in-wheel motors are changed independently so that functions of power assisted steering is realized. And the function of active steering is realized by the addition steering angle provided by the in-wheel motors. Figure 1 shows the structure of DSS. The DSS works efficiently by coupling control of in-wheel motors. When the driver turns the steering wheel, the torque and angle signal measured from the torque sensor and angle sensor are passed to the ECU. In order to achieve the DSS, the ECU combined speed, yaw rate and lateral acceleration and other signals to determine the driver's steering intention and the ideal steering torque, so that the left and right wheel motor output different torques and speeds through the different instructions[8, 9].

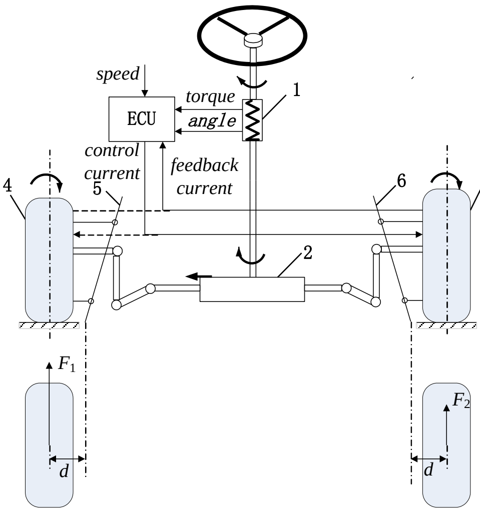

Fig. 1. The structure of the DSS
1. Torque sensor 2. Pinion and rack mechanism 3, 4. In-wheel motors 5, 6. Kingpins

3. COLLABORATIVE OPTIMIZATION (CO)

The objective of the steering system optimization is to ensure optimum steering road feel, but the vehicle performance requirements must be taken into account. In this paper, the collaborative optimization method is applied to optimize steering system parameters. The steering road feel was defined as the system-level problem, and the ergonomics, automobile security and the steering economy were taken as subsystem problem.

The basic framework of CO is shown in Fig. 2 [10, 11]. The top level is the main system optimizer whose task is to optimize the multidisciplinary variables so that the multidisciplinary constraint J^* is satisfied and system objective F is minimized. The shared design variables and coupled state variables of each subsystem are adjusted according to the main system level equality constraint, where x_s is the shared design variable; L_x is the local design variable and c is the subsystem constraint.

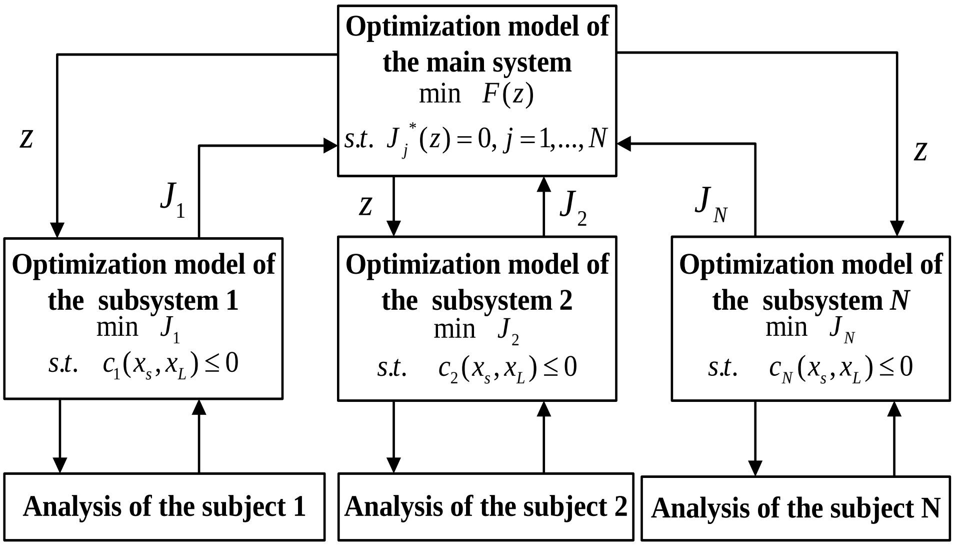

Fig. 2. The framework of CO

The optimization objective of subsystem is to minimize the difference between the optimization results of subsystem and optimization targets provided by main system, which is depicted as

$$J_j = \left|x_s - z_j^{\ s}\right|^2 + \left|y - z_j^{\ c}\right|^2 \tag{1}$$

where z^s is the system design variables; z^c is the system coupled variable and y is the couple state variable.

4. OPTIMIZATION MODEL

In the process of DSS designing, the contradiction between the steering road feel and the steering sensitivity cannot be avoided. The mechanic character cannot be ignored when ensuring steering road feel and steering sensitivity. Besides, the economy is always an important problem in vehicle design. The steering economy (the output power P of in-wheel motor) is taken as the optimization objective, and the steering road feel, steering sensitivity and the mechanic character of steering sensors are taken as the subsystems to build the CO model of the DSS.

a) Optimization model of the main system

The surrogate model of steering power consumptions is built, taking the heavy computation into consideration. The experiment variables are determined by optimal Latin hypercube experimental design. The second order response surface mode is depicted as [12, 13]:

$$\begin{aligned} P = {} & -0.0372 - 0.0175B_e + 3.4994\times10^{-7}C_1 + 1.887\times10^{-6}C_2 - 72.6870J_e - 4.2687k_1 + 0.1052K_m - 0.0003K_s + 0.0078 \\ & \cdot B_e^{\ 2} + 2.4705\times10^{-12}C_1^{\ 2} - 9.4068C_2^{\ 2} + 25095.4605J_e^{\ 2} - 5.5341\times10^{-11}k_1^{\ 2} - 0.1739K_m^{\ 2} + 2.5765K_s^{\ 2} - 2.6751\times10^{-7}B_eC_1 \\ & + 3.1165\times10^{-7}B_eC_2 + 13.2854B_eJ_e + 8.1849\cdot B_ek_1 + 0.0151B_eK_m + 5.3486\times10^{-5}B_eK_s - 1.7151\times10^{-11}\cdot C_1C_2 - 0.0005C_1J_1 \\ & -2.5454\times10^{-12}C_1k_1 + 1.15105\times10^{-6}\cdot C_1K_m + 9.9916\times10^{-10}C_1K_s + 0.0004C_2J_e + 1.6439\times10^{-11}\cdot C_2k_1 - 1.1097\times10^{-6}C_2K_m \\ & -5.0049\times10^{-10}C_2K_s + 0.0012\cdot J_ek_1 + 126.6274J_eK_m \end{aligned} \tag{2}$$

where C_1 is rolling angle stiffness of the front suspension; C_2 is rolling angle stiffness of the rear suspension.

b) Optimization model of road feel

In order to reduce the influence of road disturbance and improve the steering portability, the steering power in a certain frequency domain should be as small as possible. The subsystem model is built as

$$\begin{cases} Minimize: \\ R_s = (1 - B_e / B_e')^2 + (1 - J_e / J_e')^2 + (1 - K_m / K_m')^2 \quad + (1 - K_s / K_s')^2 \\ s.t.\ Sc = f\left(\mathbf{Z_s}\right) = \dfrac{1}{2\pi\omega_0} \displaystyle\int_0^{\omega_0} \left| E\left(j\omega\right) \right|^2 d\omega \\ 0.5 \le B_e' \le 6, 0.0001 \le J_e' \le 0.001, 0.2 \le K_m' \le 5, Sc < Sc_0 \end{cases} \tag{3}$$

where Sc_0 is the initial value steering feel; $\mathbf{Z_s} = \{B_e', J_e', K_m', K_s'\}$.

The objective function $f\left(\mathbf{Z_s}\right)$ presents the mean value of steering feel in the frequency domain $(0, \omega_0)$ where road information is most obvious. ω_0 is the biggest frequency of useful road information, which is 40Hz in the optimization. When the vehicle is in low speed, the road feel should be small, so $f\left(\mathbf{Z_s}\right)$ is relatively small.

c) Optimization model of steering sensitivity

$g\left(\mathbf{Z_f}\right)$ is the mean value of steering sensitivity in a certain frequency domain. The subsystem model is depicted as

$$\begin{cases} Minimize: \\ R_f = (1 - B_e / B_e'')^2 + (1 - J_e / J_e'')^2 + (1 - K_m / K_m'')^2 \quad + (1 - K_s / K_s'')^2 + (1 - C_1 \big/ C_1'')^2 + (1 - C_2 \big/ C_2'')^2 \\ +(1 - k_1 \big/ k_1'')^2 \\ s.t.\ Fl = f\left(\mathbf{Z_f}\right) = \dfrac{1}{2\pi\omega_0} \displaystyle\int_0^{\omega_0} \left| \dfrac{\omega_r\left(j\omega\right)}{\theta_c\left(j\omega\right)} \right|^2 d\omega \\ 0.5 \le B_e'' \le 6, 0.0001 \le J_e'' \le 0.001, 0.2 \le K_m'' \le 5,\ 20000 < C_1'' < 70000, 20000 < C_2'' < 70000, \\ -80000 < k_1'' < -20000, Fl > Fl_0 \end{cases} \tag{4}$$

where Fl_0 is the steering sensitivity before optimization; $\mathbf{Z_f} = \{B_e'', J_e'', K_m'', K_s'', C_1'', C_2'', k_1''\}$; ω_0 is also 40Hz here; $g\left(\mathbf{Z_f}\right)$ should be large when the vehicle is in low speed.

d) Optimization model of steering sensor

The torsion bar is an important stressed part in the torque and angle sensor. The stiffness of torsion bar is closely related to the stiffness of entire steering system and measurement accuracy of sensor. If the torsion bar stiffness is too big, the measurement accuracy will be low. If it is too small, the stiffness of steering system will be small which will cause understeer. Therefore, the stiffness of torsion bar must be chosen carefully. The design variables which have great influence on T_r are chosen through bode diagram. The subsystem model is built as follows:

$$\begin{cases} Minimize: \\ R_s = (1 - B_e / B_e''')^2 + (1 - K_m / K_m''')^2 + (1 - K_s / K_s''')^2 \quad + (1 - k_1 \big/ k_1''')^2 \\ s.t.\ \ \Delta\theta = f(\mathbf{Z}_t) = T_s / K_s = (T_c - T_r) / K_s = (T_c - T_r) / K_s = \left\{ T_c - \dfrac{2dk_1}{n} \left(\beta + \dfrac{a\omega_r}{v_m} + E_1\varphi - \delta \right) \right\} / K_s \\ 0.5 \le B_e''' \le 6, 0.2 \le K_m''' \le 5, -5° \le \Delta\theta \le 5°, -80000 < k_1''' < -20000 \end{cases} \tag{5}$$

where the angle of torsion bar should be $-5° \le \Delta\theta \le 5°$, considering the stiffness and accuracy of sensor. T_r is the torque about the steering out shaft produced by tire resistance torques; β is slip angle; φ is rolling angle; δ is steering angle; a is displacement from vehicle mass center to front axle; E_1 is front roll steer coefficient; $\boldsymbol{Z}_t = (B_e''', K_m''', K_s''', k_1''', n''')$.

5. OPTIMIZATION ALGORITHM

a) Particle Swarm Optimization (PSO)

PSO was introduced by American social psychologists James Kennedy and electrical engineer Russell Eberhart [14]. The basic idea is to simulate the behavior of birds group. Based on the biotic group model and concepts of "swarm" and "evolution", each individual (particle) moves according to the fitness values. PSO regards individuals in the swarm as particles without quality and volume in N-dimensional search space. Each particle moves at a certain speed adjusted by its experiment as well as swarm's experiment. Most PSO variants are improved from the PSO with inertia weight algorithm which is called standard PSO algorithm.

$X_i = (X_{i1}, X_{i2}, \cdots, X_{in})$ is the present position of particle i; $V_i = (V_{i1}, V_{i2}, \cdots, V_{in})$ is the present speed of particle i; $P_i = (P_{i1}, P_{i2}, \cdots, P_{in})$ is the best known position of particle i, which is the best individual position (pbest) with the best fitness value; $P_g = (P_{g1}, P_{g2}, \cdots, P_{gn})$ is the best known position of swarm (gbest).

The evolution equations of standard PSO algorithm are depicted as

$$V_{ij}(t+1) = \omega V_{ij}(t) + c_1 r_{1j}(t)(P_{ij}(t) - X_{ij}(t)) + c_2 r_{2j}(t)(P_{gj}(t) - X_{ij}(t)) \tag{6}$$

$$V_{ij}(t+1) = X_{ij}(t) + V_{ij}(t+1) \tag{7}$$

where subscript i presents the particle i; subscript j presents the *j*-th dimension; t presents the *t*-th generation; c_1, c_2 are the speed constants; ω is the inertia weight; $r_{ij}(t) \sim U(0,1), r_{2j}(t) \sim U(0,1)$ are the random numbers which are independent with each other; ω can be a fixed value or linear varying value; c_1, c_2 are usually chosen in (0, 2].

In order to ensure the convergence of algorithm, each particle must converge at position P, due to the tracing ability of particle and the aggregation of swarm. In standard PSO algorithm, the particle moves along track with limited speed. Therefore, the search space of particle cannot cover the entire feasible space. The Global optimal solution cannot be achieved by standard PSO algorithm, which is the biggest weakness of standard PSO algorithm.

b) Hybrid particle swarm optimization based on parallel directional turbulence (HPSO-PDT)

The core concept of HPSO-PDT is parallel directional turbulence. $m \times n$ matrix X is position of swarm; m is particle number; n is independent variable number of the objective functions, as well as the dimensions of search space; P_g is the best known position of the swarm. When the swarm is premature, each dimension of P_g mutates to a new swarm based on PDT [15, 16].

Parallel directional criterion of particle is determined by directional information matrix B. B is a m×n logic matrix. $z = \min(m, n)$; A is the *z*-dimension integer vector, $A \in [1, 2, \cdots, n]$. The elements of B meet the following equations

$$\begin{cases} \begin{cases} B_{i,j} = 1, j = A(i) \\ B_{i,j} = 0, else \end{cases}, m \le n \\ \begin{cases} B_{i,j} = 1, \mathrm{mod}(i,n) + 1 = A(j) \\ B_{i,j} = 0, else \end{cases}, m > n \end{cases} \tag{8}$$

where $i \in [1, 2, \cdots, n]$; $j \in [1, 2, \cdots, n]$; mod presents modulo operator. The directional information matrix B decides the movement of each particle.

The mutation search space of particle narrows linearly, which improves the global search in early iterate process and disturbance mutation in late iterate process. The boundaries of mutation space are

$$\begin{cases} q_L = \lambda q_L^0 + (1-\lambda) P_g \\ q_U = \lambda q_U^0 + (1-\lambda) P_g \end{cases} \tag{9}$$

where q_U and q_L are the upper and lower boundary, respectively; q_U^0 and q_L^0 are the upper and lower boundary of initial search space, respectively; λ is the constriction factor.

$$\lambda = 1 - \frac{t}{MAX} \tag{10}$$

where t is the current iterate number; MAX is the biggest iterate number.

The m×n matrix s is built by Logistic chaotic mapping from an 0-1 uniform distribution row victor with n-dimension. The mutation matrix γ of particle positions is expressed as

$$\gamma_{i,j} = q_{L,j}(1 - S_{i,j}) + q_{U,j} S_{i,j} \tag{11}$$

The mutation of particles meets the following equation

$$\mathrm{X}_{i,j}^t = \gamma_{i,j} B_{i,j} + P_{g,j}(1 - B_{i,j}) \tag{12}$$

In order to improve directional turbulence, the current swarm speed $V_{i,j}^t$ is set to zero; the current swarm position is set to the best known position of particle $P_{i,j}^t$.

When the element of directional information matrix B is 1, the value of corresponding element of X is the same as the value of mutation matrix γ. So each particle just moves in one dimension and turbulence of HPSO-PDT only occurs in z directions in each mutation.

Traditional optimal algorithm such as sequence quadratic programming is combined with parallel directional turbulence to increase the convergence speed in global optimization.

When swarm is premature, the process of disturbance mutation of HPSO-PDT is as follows:

The new best position P_g is obtained by local search using sequence quadratic programming with the current best position as start point. The new swarm X is produced from Eqs. (8)-(12) with P_g. After 3 times of PSO iteration, if new P_g is updated, iteration continues and above steps repeats when swarm is premature; If P_g is invariant, *z*-dimension integer vector A is rebuilt, mutation proceeds until new P_g is obtained.

6. OPTIMIZATION RESULTS

Global design variables are transferred from main system to subsystems. Because of its strong search capability, the particle swarm algorithm is applied in global optimization. The directional heuristic search algorithm (DHS) is applied in subsystem optimization in order to ensure the coordination between main system and subsystem design variables. The Pareto optimization solution sets of road feel, steering sensitivity and steering economy are shown as Fig. 3.

There are 216 Pareto optimization solution sets in the collaborative optimization based on particle swarm algorithm, far more than the general optimization algorithm. This shows that it is easier for collaborative optimization to find the best solution based on particle swarm algorithm.

The iterative process of steering economy, road feel and steering sensitivity of collaborative optimization based on particle swarm algorithm are shown as Figs. 4-6, respectively.

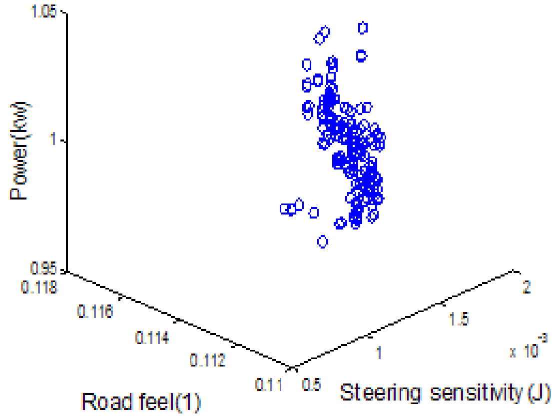

Fig. 3. Pareto optimization solution sets

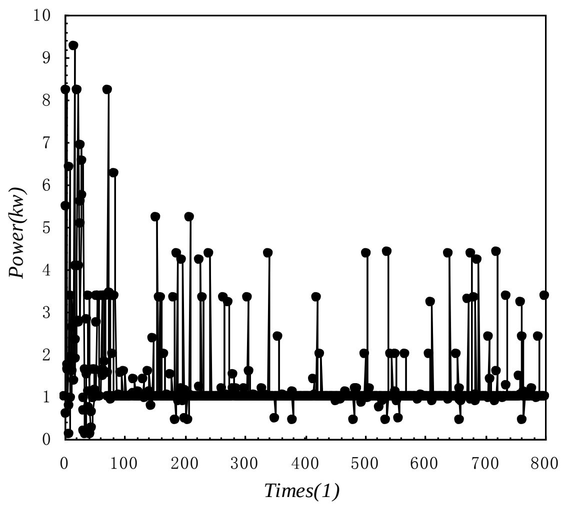

Fig. 4. The iterative process of steering economy

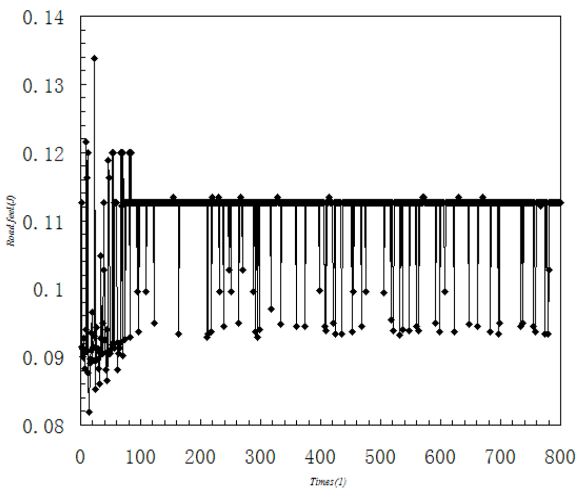

Fig. 5. The iterative process of road feel

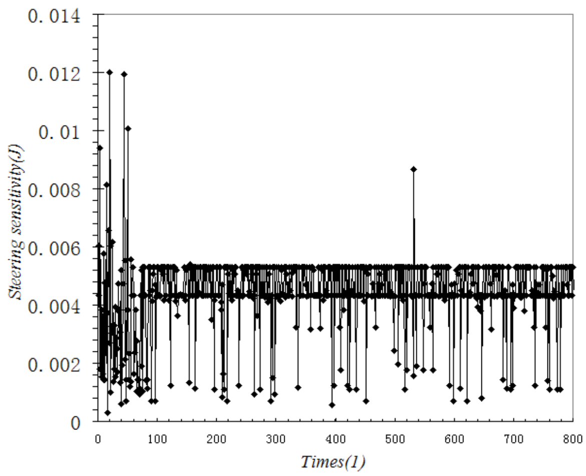

Fig. 6. The iterative process of steering sensitivity

It can be seen from Figs. 4-6 that the performance indexes of collaborative optimization based on particle swarm algorithm have faster convergence speed. Especially for the road feel, there is less oscillation during the iterative process. The local design variables are optimized independently in each subsystem under the collaborative optimization based on particle swarm algorithm, which is different from the general collaborative optimization. The optimal values of general collaborative optimization and collaborative optimization based on particle swarm algorithm are shown as Table 1.

Table 1. Optimal values

Design variables	Initial values	Optimal values of general CO	Optimal values of CO based on particle swarm algorithm
Be(N·m/(rad·s))	2.63	1.96	2.76
C1(N·m/rad)	64896	42789	20452
Je(kg·m2)	1.07×10^{-4}	8.44×10^{-4}	8.85×10^{-4}
Km(A/(N·m))	2.23	3.21	2.27
Ks(N·m/rad)	237	209	223
n	17	18	16
k_1 (N/m)	-56963	-57286	-48450
Ka	2.5	1.91	1.47
Sc	0.15	0.11	0.11
Fl	9.03×10^{-4}	9.29×10^{-4}	1.05×10^{-3}
P(kw)	1.98	1.20	0.98

It can be seen from Table 1 that under the general CO, the optimal value of road feel is 0.11J, 27% lower than initial value; the steering sensitivity is 9.29e-4J, increased by 2.9% and energy consumption is 1.2kw, reduced by 39%. Under the CO based on particle swarm algorithm, the road feel is 0.11J, reduced by 27%; the steering sensitivity is 1.05e-3J, increased by 16% and energy consumption is 0.98kw, reduced by 51%. The results show that optimal values of each performance index under the CO based on particle swarm algorithm are better than the general CO.

The Bode diagrams of road feel and steering senstivity under general CO and CO based on particle swarm algorithm are shown in Figs. 7-8.

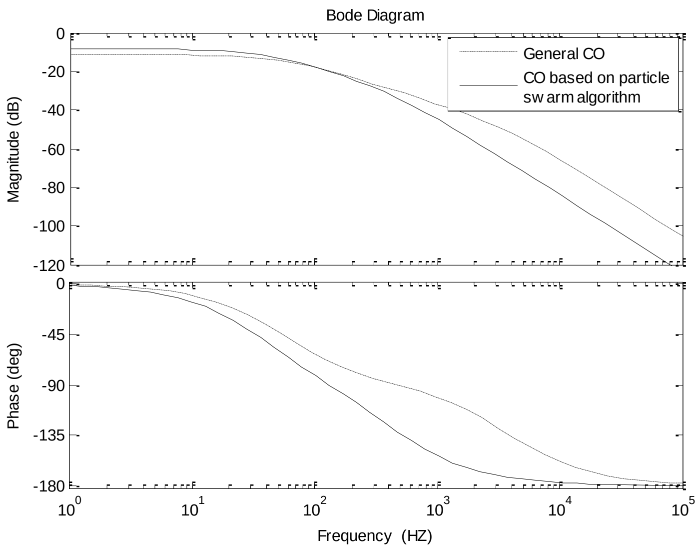

Fig. 7. Bode diagram of road feel

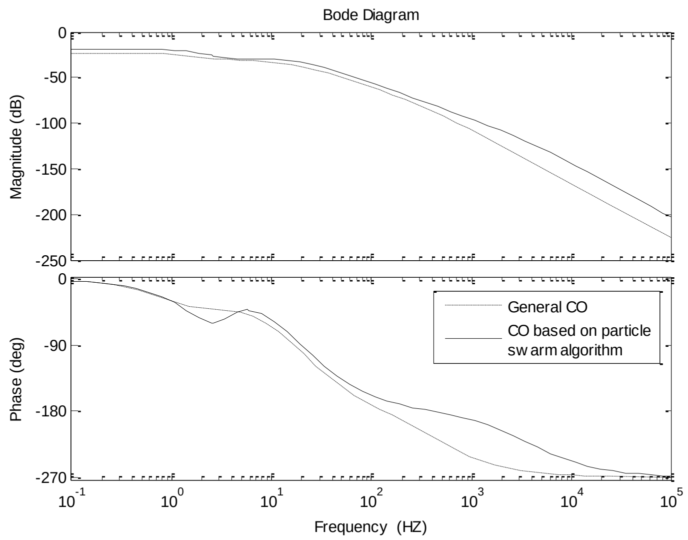

Fig. 8. Bode diagram of road feel of steering sensivity

It can be seen from Figs. 7-8 that the comparisons between the conventional collaborative optimization and the one based on particle swarm optimization show that the static error of steering feel decreases, response speed increases and band width widens, so the system with particle swarm optimization responds more accurately. The optimal solution of steering feel, steering sensitivity and steering economy satisfying the constraints of DSS is derived by the collaborative optimization based on particle swarm optimization so that the performance of steering portability and steering feel are combined perfectly.

7. CONCLUSION

a) The DSS for the in-wheel electric vehicle is introduced, which not only improves the steering portability and road feel, but combines the active safety with steering economy.
b) A collaborative optimization model of the DSS is built, with steering economy as the main system optimization goal, steering road feel, steering sensitivity and torque sensor performance as the subsystem optimization goals. The main system is optimized by the particle swarm algorithm and the subsystems are optimized by the directional heuristic search algorithm.
c) The optimization shows that the CO based on particle swarm algorithm provides a faster convergence speed and more optimization solution sets with considering the coupling relationship between different disciplines. The comprehensive performance of motorized wheels electric vehicle is improved. With satisfying steering feel, good robust performance and steering stability being the control objectives, the models of the novel AFS system are set up, and the time-delay H_∞ controller for the novel AFS system is designed.

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